Lesson Overview

LESSON 18

Compare Fractions

CCSS Focus

Domain
Number and Operations—Fractions

Cluster
A. Extend understanding of fraction equivalence and ordering.

Standard
4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Additional Standard
4.NF.A.1 (See Standards Correlations at the end of the book for full text.)

Standards for Mathematical Practice (SMP)
SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

4 Model with mathematics.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 363m to see how every lesson includes these SMPs.

Lesson Objectives

Content Objectives
- Use symbols ($>$, $<$, $=$) to compare fractions with different numerators and different denominators.
- Recognize that fractions with different denominators and the same numerators represent different values.
- Use common denominators and benchmark fractions to compare fractions with different denominators.
- Recognize that to compare two fractions both must refer to the same whole.

Language Objectives
- Write fraction comparison statements using the symbols $>$, $<$, and $=$.
- Draw area models to compare two fractions.
- Orally explain how comparing both a fraction greater than $\frac{1}{2}$ and a fraction less than $\frac{1}{2}$ to $\frac{1}{2}$ can be used to determine which fraction is greater.

Prerequisite Skills
- Represent fractions with denominators 2, 3, 4, 6, or 8 using a number line or visual models.
- Identify, generate, and explain equivalent fractions.
- Express whole numbers as fractions.
- Compare fractions with the same numerators or denominators.

Lesson Vocabulary
- **benchmark fraction** a common fraction that you might compare other fractions to. For example, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are often used as benchmark fractions.
- **common denominator** a number that is a common multiple of the denominators of two or more fractions.

Review the following key terms.

- **compare** to decide if numbers, amounts, or sizes are greater than, less than, or equal to each other.
- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **greater than symbol ($>$)** a symbol used to compare two numbers when the first is greater than the second.
- **less than symbol ($<$)** a symbol used to compare two numbers when the first is less than the second.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.
- **unit fraction** a fraction with a numerator of 1. Other fractions are built from unit fractions.

Learning Progression

In Grade 3 students used models to compare two fractions with the same numerator or the same denominator by reasoning about their size.

In Grade 4 students extend their understanding of fractions to compare two fractions with different numerators and different denominators. Emphasis is placed on understanding that a comparison only makes sense if the two fractions have the same-sized wholes. In this lesson students use models to compare fractions by using common numerators or denominators. Students also use benchmark fractions to compare fractions. They record comparisons using the symbols $>$, $<$, and $=$.

In Grade 5 students will apply their understanding of fraction comparison when they learn to compare decimals.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Equivalent Fractions

Comparing Fractions
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 383–384

SESSION 2
Develop
45–60 min

Using Common Numerators and Denominators
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model Its *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 389–390

Fluency
Using Common Numerators and Denominators

SESSION 3
Develop
45–60 min

Using a Benchmark to Compare Fractions
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model It & Solve It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 395–396

Fluency
Using a Benchmark to Compare Fractions

SESSION 4
Refine
45–60 min

Comparing Fractions
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 1 set of fraction tiles
Activities	*Per student:* 1 set of fraction tiles *Per pair:* scissors, colored pencils, index cards *Activity Sheets:* Number Lines, 1-Centimeter Grid Paper
Math Toolkit	fraction circles, fraction tiles, number lines, fraction bars, index cards, hundredths grids, tenths grids
Digital Math Tools	Fraction Models, Number Line

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons
Grade 3
- Lesson 23 Find Equivalent Fractions
- Lesson 25 Use Symbols to Compare Fractions

RETEACH

Tools for Instruction
Grade 3
- Lesson 23 Find Equivalent Fractions
- Lesson 25 Use Symbols to Compare Fractions

Grade 4
- Lesson 18 Compare Fractions

REINFORCE

Math Center Activities
Grade 4
- Lesson 18 Use Fraction Vocabulary
- Lesson 18 Comparing Fractions

EXTEND

Enrichment Activity
Grade 4
- Lesson 18 Colorful Quilts

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*
Grade 4
- Compare Fractions

Learning Game
- *Bounce*

Connect to Family, Community, and Language Development

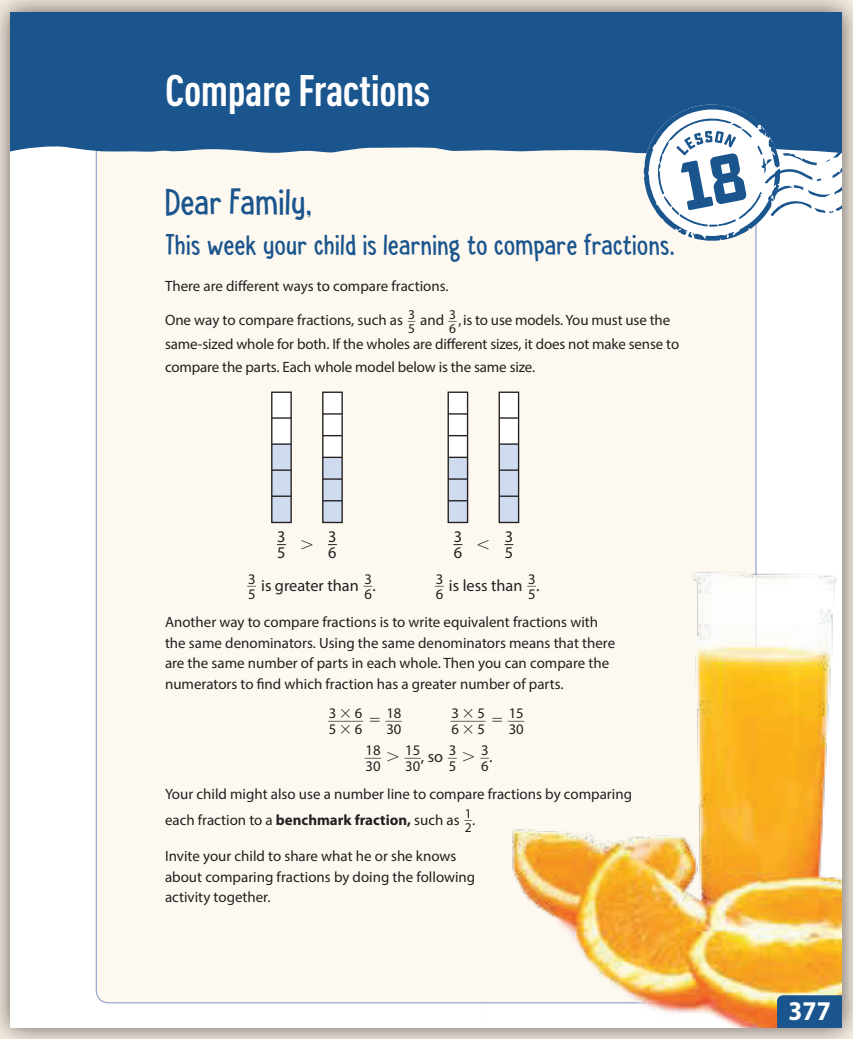
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Compare Fractions

LESSON 18

Dear Family,

This week your child is learning to compare fractions.

There are different ways to compare fractions.

One way to compare fractions, such as $\frac{3}{5}$ and $\frac{3}{6}$, is to use models. You must use the same-sized whole for both. If the wholes are different sizes, it does not make sense to compare the parts. Each whole model below is the same size.

$\frac{3}{5} > \frac{3}{6}$ $\frac{3}{5}$ is greater than $\frac{3}{6}$.

$\frac{3}{6} < \frac{3}{5}$ $\frac{3}{6}$ is less than $\frac{3}{5}$.

Another way to compare fractions is to write equivalent fractions with the same denominators. Using the same denominators means that there are the same number of parts in each whole. Then you can compare the numerators to find which fraction has a greater number of parts.

$\frac{3 \times 6}{5 \times 6} = \frac{18}{30}$ $\frac{3 \times 5}{6 \times 5} = \frac{15}{30}$

$\frac{18}{30} > \frac{15}{30}$, so $\frac{3}{5} > \frac{3}{6}$.

Your child might also use a number line to compare fractions by comparing each fraction to a **benchmark fraction,** such as $\frac{1}{2}$.

Invite your child to share what he or she knows about comparing fractions by doing the following activity together.

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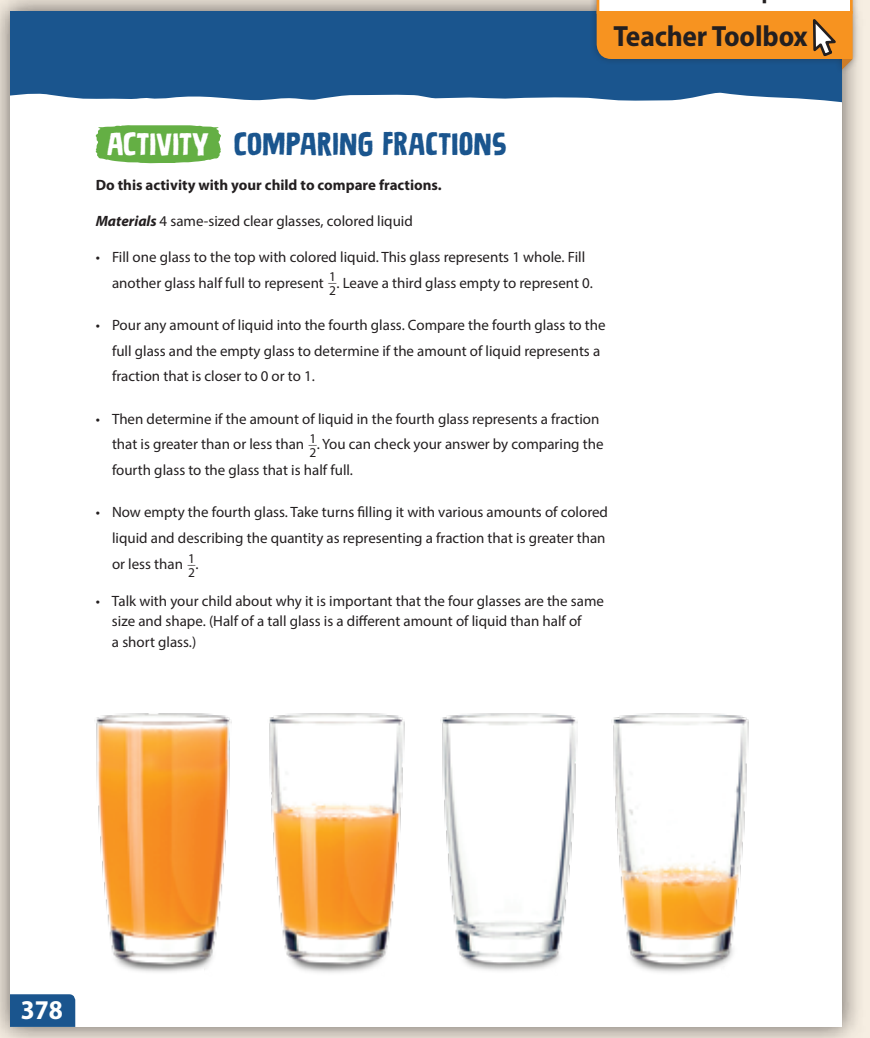

ACTIVITY COMPARING FRACTIONS

Do this activity with your child to compare fractions.

Materials 4 same-sized clear glasses, colored liquid

- Fill one glass to the top with colored liquid. This glass represents 1 whole. Fill another glass half full to represent $\frac{1}{2}$. Leave a third glass empty to represent 0.
- Pour any amount of liquid into the fourth glass. Compare the fourth glass to the full glass and the empty glass to determine if the amount of liquid represents a fraction that is closer to 0 or to 1.
- Then determine if the amount of liquid in the fourth glass represents a fraction that is greater than or less than $\frac{1}{2}$. You can check your answer by comparing the fourth glass to the glass that is half full.
- Now empty the fourth glass. Take turns filling it with various amounts of colored liquid and describing the quantity as representing a fraction that is greater than or less than $\frac{1}{2}$.
- Talk with your child about why it is important that the four glasses are the same size and shape. (Half of a tall glass is a different amount of liquid than half of a short glass.)

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Goal

The goal of the Family Letter is to encourage students and family members to compare fractions.

- Students and family members explore how to compare fractions by using models and equivalent fractions with common denominators. Students are introduced to the term *benchmark fraction* as a strategy for comparing fractions.

Activity

Students and family members compare the amounts of liquid in 4 glasses that are the same shape and size. Three glasses are filled with varying amounts of colored liquid to represent 1, $\frac{1}{2}$, and 0. The fourth glass is filled with a random amount to compare to the amounts in the other glasses.

Math Talk at Home

Explain to students that the glasses used in the Family Letter activity need to be the same size and shape. If students do not have the materials at home to do the activity, encourage them to compare fractions in casual conversations with family members using common same-sized items they have at home.

Conversation Starters Below are additional questions students can write in their Family Letter or math journal to engage family members:

- *If you have a glass that is $\frac{1}{2}$ full, and I have a glass that is $\frac{3}{4}$ full, who has the greater amount?*
- *If I have read $\frac{1}{3}$ of a book and you have read $\frac{1}{2}$ of the same book, who has more of the book left to read?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- Many students will be familiar with granola bars, but there may be some who cannot eat them because of food allergies. Encourage students to substitute food items they are familiar with in the problem to make it relevant to them. For example, instead of same-sized granola bars, students may visualize same-sized carrots or celery sticks. Remind students that the food items they use in the word problem must be the same size for comparing fractions.

Session 2 **Use with *Apply It* problem 9.**

- Ask students if they know where tomatoes or peppers originate from or if they have grown a tomato or pepper plant. Some students may respond, "the grocery store," or may have never considered where tomatoes or peppers come from. Show students pictures of tomato and pepper plants. Ask students what other fruits and vegetables they are familiar with. Possible responses include cucumbers, squash, beans, peas, okra, corn, and grapes. If there are regional fruits or vegetables students may be more familiar with, substitute these in the word problem.

Session 3 **Use with *Additional Practice* problems.**

- As students read the problems, encourage them to think of scenarios to make them more interesting and relevant. Model this strategy for students using problem 1. In this problem, students are asked to compare $\frac{9}{10}$ and $\frac{3}{2}$. Say: *My coach asked me to run $\frac{9}{10}$ of the track on Monday. On Tuesday, she asked me to run $\frac{3}{2}$ of the track. On which day did I run a lesser distance?* Have students share their scenarios for the problems with partners.

Session 4 **Use with *Apply It* problem 6.**

- Ask students if they know what a trombone is and to give examples of when they have seen one being played. Explain that a trombone is a brass instrument. To become a proficient trombonist, a person practices the trombone for many hours. In *Apply It* problem 6, the student practices his trombone $\frac{2}{6}$ of an hour. Ask students to share with the class instruments they play or would like to play. As students read the word problem, encourage them to substitute other instruments they are familiar with.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Connect It*.

Levels 1–3	Levels 2–4	Levels 3–5
Speaking/Reading Read ***Connect It*** problem 2b to students. Explain that to understand the problem, students need to understand the terms in it. Write the following terms and symbols on sentence strips: *denominator*, $>$, $<$, $=$, and *equivalent fractions*. Display the term *denominator*. Ask students to define *denominator* to partners before sharing definitions with the group. Select one of the definitions to write on a sentence strip. Continue the process with the remaining terms. Shuffle the sentence strips. Ask students to read and match the sentence strips. Reread ***Connect It*** problem 2b. Have students restate the information in their own words.	**Writing/Reading** Choral read ***Connect It*** problem 2b. Explain that to understand the problem, students need to understand the terms in it. Have students form pairs. Ask students to write the following terms and symbols on index cards and then work with their partners to write the definitions on other cards: *compare*, *fractions*, $>$, $<$, $=$, and *equivalent fractions*. Have pairs shuffle their cards and exchange them with another group. Ask them to match the terms with the definitions. Reread ***Connect It*** problem 2b with students. Ask students to restate the information in their own words to their partners.	**Speaking/Writing** Have students read ***Connect It*** problem 2b with partners. Have partners identify the terms and symbols in the problem that they think are important for understanding the problem. Then ask students to work with their partners to explain why they think the terms and symbols they chose are important. Ask partners to write the terms, symbols, and definitions on index cards, shuffle them and then exchange them with other pairs to match. Have students reread ***Connect It*** problem 2b with partners and have them restate the information in their own words.

LESSON 18

SESSION 1 Explore

Purpose In this session, students draw on their experience comparing fractions in order to compare two fractions with the same numerators and different denominators. They share models to explore how various solution methods are based on the number of equal parts in each whole and the sizes of the parts. They will look ahead to think about comparing fractions with different denominators by using equivalent fractions with a common denominator.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles

Why Support students' facility with comparing fractions with the same numerators.

How Have students use fraction tiles to compare $\frac{3}{5}$ and $\frac{3}{6}$.

Compare $\frac{3}{5}$ and $\frac{3}{6}$.

Which fraction is greater?

Solution
$\frac{3}{5}$ is greater than $\frac{3}{6}$.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that the granola bars are the same size and that Adriana eats 2 out of 4 equal parts of her bar while June eats 2 out of 5 equal parts of her bar.

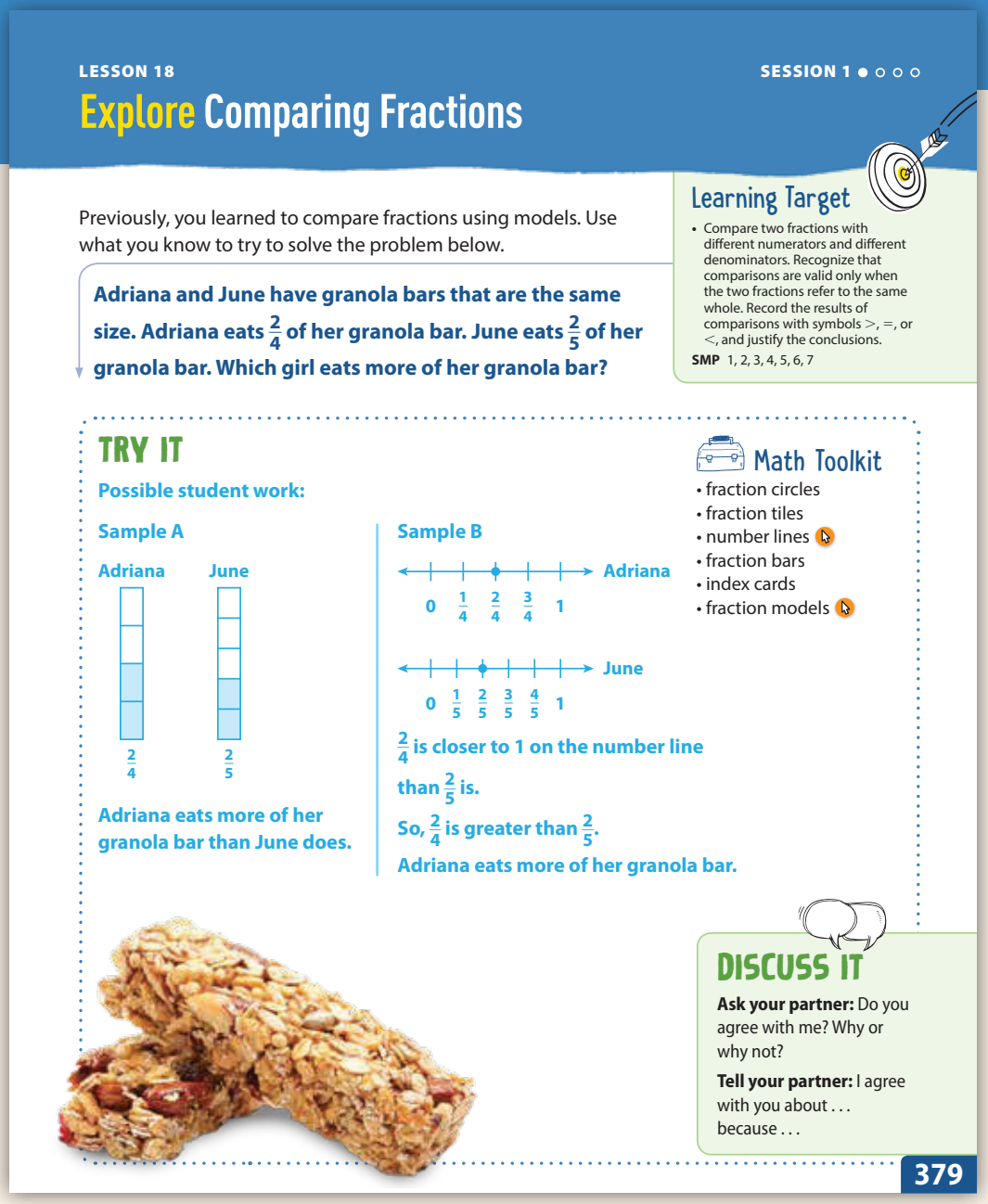
DISCUSS IT

Support Partner Discussion

To reinforce the units of fourths and fifths, encourage students to use the terms *fourths* and *fifths* as they talk together.

Look for, and prompt as necessary, for understanding of:

- both wholes as the same size
- 4 as the number of equal parts in one whole
- 5 as the number of equal parts in the other whole
- 2 as the number of parts considered in each whole



Common Misconception Look for students who think that $\frac{2}{5}$ is greater than $\frac{2}{4}$ because 5 is greater than 4. As students present solutions, have them explain how the sizes of the equal parts in the granola bars compare to each other.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction circles or fraction tiles modeling two fourths and two fifths
- drawings of area models showing two fourths and two fifths
- labeled number lines showing the locations of two fourths and two fifths
- writing equivalent fractions to compare two fourths and two fifths

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the number of equal parts in each whole? the number of parts considered?*

Listen for The models are divided into 4 and 5 equal parts or have denominators of 4 and 5. The models have 2 parts shaded, or counted, or have numerators of 2.

CONNECT IT

1 LOOK BACK

Look for understanding that Adriana eats more and that $\frac{2}{4}$ is greater than $\frac{2}{5}$ because the size of each of Adriana's two $\frac{1}{4}$-pieces is greater than the size of each of June's two $\frac{1}{5}$-pieces.

Hands-On Activity

Use fraction tiles to compare fractions with the same numerator.

If . . . *students are having difficulty comparing fractions with the same numerators,*

Then . . . *use this activity to provide a more concrete experience in reasoning about the size of the unit fractions that make up each fraction.*

Materials For each student: 1 set of fraction tiles

- Have students compare $\frac{2}{3}$ and $\frac{2}{8}$. Tell them to trace around the one-whole tile two times, so they have an outline of one whole for each fraction.
- Have students identify the unit fractions for $\frac{2}{3}\left[\frac{1}{3}\right]$ and $\frac{2}{8}\left[\frac{1}{8}\right]$ and compare the sizes of the fraction tiles for $\frac{1}{3}$ and $\frac{1}{8}$ to find which covers a larger area. $\left[\frac{1}{3}\right]$ Say: *Each third is larger than each eighth, so $\frac{2}{3}$ is greater than $\frac{2}{8}$.*
- Have students use the tiles and the one-whole outlines to build the fractions $\frac{2}{3}$ and $\frac{2}{8}$ to see that $\frac{2}{3}$ is greater than $\frac{2}{8}$.
- Repeat for additional fractions with the same numerators, such as $\frac{3}{6}$ and $\frac{3}{4}$, and $\frac{4}{5}$ and $\frac{4}{10}$.

2 LOOK AHEAD

Point out that the same symbols used to compare whole numbers are used to compare fractions. Review the meanings of the symbols $>$, $<$, and $=$. Also point out that you can compare fractions with different denominators by rewriting one or both of them to have a common denominator. Ask a volunteer to restate the definition of the term *common denominator*. Students will spend more time learning about the concept of common denominators in the Additional Practice.

Students should be able to use their work with equivalent fractions to rewrite the given fractions with a common denominator.

CONNECT IT

1 LOOK BACK

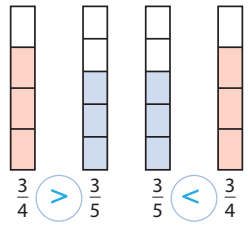
Who eats more of her granola bar, Adriana or June? Explain.

Adriana eats more. Possible explanation: Both girls eat the same number of pieces, but Adriana's two pieces are larger than June's two pieces.

2 LOOK AHEAD

Deciding who eats more of her granola bar means comparing the fractions $\frac{2}{4}$ and $\frac{2}{5}$. To compare fractions, you must use the same-sized whole.

a. Suppose you have two more granola bars that are the same size. Compare the fractions $\frac{3}{4}$ and $\frac{3}{5}$ using the area models to know who ate more. Use $>$, $<$, or $=$ to compare, just as with whole numbers.



$\frac{3}{4} > \frac{3}{5}$ $\quad \frac{3}{5} < \frac{3}{4}$

b. You can use equivalent fractions to compare fractions with different denominators. Compare $\frac{3}{4}$ and $\frac{3}{5}$. Rewrite one or both of the fractions so they have the same denominator, or a **common denominator**. Use $>$, $<$, or $=$ to compare.

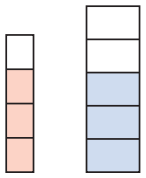
$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$ $\quad \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$

$\frac{15}{20} > \frac{12}{20}$, so $\frac{3}{4} > \frac{3}{5}$.

3 REFLECT

Suppose the granola bars were different sizes. Could you still compare $\frac{3}{4}$ and $\frac{3}{5}$ in the same way? Explain.

No. Possible explanation: Because the wholes are two different sizes, comparing parts to each other does not make sense.



Close: Exit Ticket

3 REFLECT

Look for understanding that a comparison of two fractions only makes sense when the wholes are the same size.

Common Misconception **If** students do not mention that the sizes of the wholes must be the same in order to compare the fractions, **then** have them look at the different-sized wholes in the models in problem 3 and identify how the size of the wholes differ so the parts cannot be compared.

Real-World Connection

Encourage students to think about everyday places or situations in which people might need to compare fractions. Have volunteers share their ideas. Examples include following a recipe, measuring lengths of fabric, and making carpentry measurements.

Solutions

Support Vocabulary Development

1 Ask students to use what they know about the meaning of the term *common denominator* based on what they know about the words *common* (something two or more things share) and *denominator* (the number below the line in a fraction that tells the number of equal parts in the whole). First, have students share their definitions with partners, then ask them to share their definitions with the group. Record responses on a chart. Remind students they found a common denominator for $\frac{3}{4}$ and $\frac{3}{5}$ in *Connect It* problem 2. Ask them to tell partners what they did to find a common denominator. Record responses on the chart. If students need support verbalizing the process for finding common denominators, write the fractions $\frac{1}{3}$ and $\frac{2}{4}$ and talk through the process. As students complete the graphic organizer, encourage them to refer to the information recorded on the chart. Remind them to record information in their own words. Also, encourage students to review the *Try It* and *Connect It* information to help them show examples and non-examples of common denominators.

2 If students struggle to complete the problem, have them think through the process for finding a common denominator before solving it. Provide guiding statements to help students.

- *Think about each fraction. Circle the denominators.*
- *Think about a multiple of each denominator that can be used as a common denominator. [15]*
- *Think about the numbers you multiply 3 and 5 by to get the common denominator. [5, 3]*
- *Think about multiplying 3 by 5 in the fraction $\frac{2}{3}$. You will also multiply the numerator, 2, by 5.*
- *Think about multiplying the denominator 5 by 3 and what number you will multiply the numerator, 2, by in the fraction $\frac{2}{5}$.*
- *Think about which fraction with denominator 15 is greater and which is less. Compare the original fractions.*

Supplemental Math Vocabulary

- *fraction*
- *denominator*

Name: _____ LESSON 18 SESSION 1

Prepare for Comparing Fractions

1 Think about what you know about common denominators. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

In My Own Words

When two or more fractions have the same denominator, they have a common denominator.

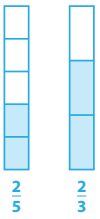
My Illustrations

common denominator

Examples

$\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ all have a common denominator of 5.

Non-Examples



2 Compare $\frac{2}{3}$ and $\frac{2}{5}$. Rewrite the fractions so they have a common denominator.

Use $>$, $<$, or $=$ to compare.

$\frac{2 \times \boxed{5}}{3 \times \boxed{5}} = \frac{10}{\boxed{15}}$ $\quad$ $\frac{2 \times \boxed{3}}{5 \times \boxed{3}} = \frac{\boxed{6}}{15}$

$\frac{10}{\boxed{15}} \boxed{>} \frac{\boxed{6}}{15}$, so $\frac{2}{3} \boxed{>} \frac{2}{5}$.

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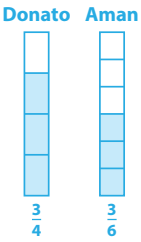
3 Solve the problem. Show your work.

Donato and Aman have bottles of juice that are the same size. Donato drinks $\frac{3}{4}$ of his juice. Aman drinks $\frac{3}{6}$ of his juice. Which boy drinks more juice?

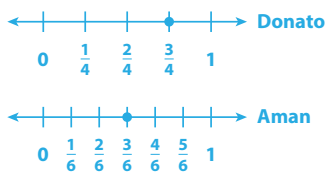
Possible student work using pictures:



Solution **Donato drinks more juice.**

4 Check your answer. Show your work.

Possible student work:



$\frac{3}{4}$ is closer to 1 on the number line than $\frac{3}{6}$ is.

So, $\frac{3}{4}$ is greater than $\frac{3}{6}$.

Donato drinks more juice.

3 Assign problem 3 to provide another look at solving a problem by comparing fractions.

This problem is very similar to the problem asking which girl eats more of her granola bar. In both problems, students compare two fractions with the same numerators, but different denominators. The question asks which boy drinks more juice.

Students may want to use fraction tiles, fraction circles, or fraction bars.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:
Students may draw diagrams to help compare the fractions. Since the bottles of juice are the same size, each fourth of a bottle is more than each sixth of a bottle, and so $\frac{3}{4}$ of a bottle is more than $\frac{3}{6}$ of a bottle. Donato drinks more juice.

Medium

4 Have students solve the problem another way to check their answer.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Speaking/Writing Use with ***Connect It*** problem 8. Ask students to brainstorm about models or strategies used to compare fractions. Encourage them to review *Try It* and *Model Its* for reminders. Make a list of responses. Point to the first model or strategy, for example, *area models*. Ask students to draw a representation for how the model or strategy is used to compare fractions. Continue the process for the remaining models or strategies. Ask: *Which model or strategy do you like best for comparing fractions?* Have students first respond verbally, then in writing: *I like to use _____ for comparing fractions.*

Levels 2–4

Speaking/Writing Use with ***Connect It*** problem 8. Ask students to brainstorm about strategies used to compare fractions. Record their responses. Have partners explain how each strategy is used to compare fractions. Listen to partner discussions. Select information to record. For example: *I heard Diana and Mila say that when you find a common denominator, you multiply both the numerator and denominator by the same number.* Ask students to identify the strategy they like best for comparing fractions and why. Provide a sentence frame: *I like to use _____ for comparing fractions because _____.*

Levels 3–5

Listening/Speaking Use with ***Connect It*** problem 8. Ask students to brainstorm about strategies used to compare fractions. Write their responses on index cards. Display a card. Have partners explain how the strategy is used to compare fractions. Shuffle the cards. Place them facedown. Call on a student to select a card. Have the rest of the students listen as the student describes how the strategy is used without naming the strategy in the description. Have students identify the strategy. Continue the process until all cards have been selected.

LESSON 18

SESSION 2 Develop

Purpose In this session, students solve a word problem that requires comparing two fractions with different numerators and different denominators. Students model the fractions in the problem either on paper or with manipulatives to determine the greater fraction. The purpose is to have students develop strategies to compare fractions with different numerators and different denominators.

Start

Connect to Prior Knowledge

Why Support students' facility with comparing fractions that have different numerators or different denominators.

How Determine whether fraction comparison statements are true or false.

Tell whether each comparison is *True* or *False*.

1. $\frac{2}{4} > \frac{2}{5}$
2. $\frac{3}{8} < \frac{7}{8}$
3. $\frac{2}{5} = \frac{2}{10}$

Solutions

1. True
2. True
3. False

Develop Language

Why Reinforce the meaning of the word *symbol*.

How Remind students that they have used symbols such as +, −, ×, and ÷ to add, subtract, multiply, and divide. They have also used symbols such as ? and a letter to stand for an unknown number in an equation. Remind students that symbols are also used to compare numbers. Display the following symbols: >, <, =. Have students tell what each symbol represents.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the fractions and the unit of measurement.

Ask *What fraction tells the weight of the grasshopper? the beetle? What unit of measurement is given for the weight of each insect?*

Develop Using Common Numerators and Denominators

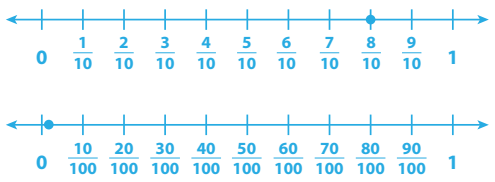
Read and try to solve the problem below.

A grasshopper weighs $\frac{2}{100}$ of an ounce. A beetle weighs $\frac{8}{10}$ of an ounce. Which weighs more?

TRY IT

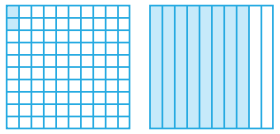
Possible student work:

Sample A



$\frac{8}{10} > \frac{2}{100}$, so the beetle weighs more.

Sample B



$\frac{2}{100}$ is much less than $\frac{8}{10}$, so the beetle weighs more than the grasshopper.

Math Toolkit
- number lines
- hundredths grids
- tenths grids
- index cards
- fraction models

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *tenths* and *hundredths* as they discuss their solutions.

Support as needed with questions such as:

- *What is another way you could have solved this problem?*
- *How do you know that your answer is reasonable?*

Common Misconception Look for students who correctly describe $\frac{8}{10}$ as greater than $\frac{2}{100}$ but confuse the comparison symbols and write < rather than >.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- hundredths grids and tenths grids shaded to represent $\frac{2}{100}$ and $\frac{8}{10}$
- labeled number lines showing the locations of the two fractions
- writing equivalent fractions to compare the two fractions
- reasoning using a benchmark fraction of $\frac{1}{2}$ to compare the two fractions

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does your model show $\frac{2}{100}$? $\frac{8}{10}$? How does your model show that the tenths and hundredths represent the same-sized whole?*

Listen for Students should recognize that accurate responses include area models divided into 100 equal parts with 2 parts shaded and into 10 equal parts with 8 parts shaded and that both models are the same size. Responses may also include number lines labeled with tenths and hundredths that show the same distance from 0 to 1.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of hundredths representing the weight of the grasshopper
- the number of tenths representing the weight of the beetle

Ask *How does each model represent hundredths? How does each model represent tenths?*

Listen for One model is divided into 100 equal parts, and the other is divided into 10 equal parts. 100 in the denominator shows the hundredths, and 10 in the denominator shows the tenths.

For the area models, prompt students to identify how the comparison of the fractions is shown.

- *Why do the models have different numbers of parts?*
- *What does the shading in each model represent?*
- *What do you observe about the size of the models?*

For a common denominator, prompt students to identify how an equivalent fraction is used.

- *What number are both 8 and 10 in $\frac{8}{10}$ multiplied by?*
- *Why is $\frac{8}{10}$ rewritten as $\frac{80}{100}$?*
- *What symbol is used to indicate equivalence?*
- *Why do you think that 100 is a better choice for the common denominator than 10?*

Explore different ways to understand comparing fractions.

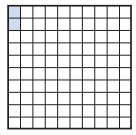
A grasshopper weighs $\frac{2}{100}$ of an ounce. A beetle weighs $\frac{8}{10}$ of an ounce. Which weighs more?

MODEL IT

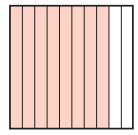
You can use models to help compare fractions.

The models show the fractions of an ounce that the grasshopper and beetle weigh.

Grasshopper



Beetle



MODEL IT

You can use a common denominator to help compare fractions.

When you compare two fractions, it helps if they have a common denominator. Fractions with the same denominator are made up of parts of the same size. The numerators tell how many of those parts each fraction has. When two fractions have the same denominator, you can compare the numerators.

Compare $\frac{2}{100}$ and $\frac{8}{10}$.

The fractions are not written with a common denominator. Find a fraction equivalent to $\frac{8}{10}$ that has a denominator of 100.

$$\frac{8 \times 10}{10 \times 10} = \frac{80}{100}$$

Now, compare the numerators of $\frac{2}{100}$ and $\frac{80}{100}$.

$80 > 2$

So, $\frac{80}{100} > \frac{2}{100}$ and $\frac{8}{10} > \frac{2}{100}$.

Deepen Understanding

Common Denominators

SMP 7 Use structure.

When discussing common denominators, prompt students to recognize that rewriting one of the fractions makes it easier to compare the two fractions.

Ask *Why do you multiply the denominator of $\frac{8}{10}$ by 10?*

Listen for You want to get a fraction equivalent to $\frac{8}{10}$ that has the same denominator as $\frac{2}{100}$.

Ask *Why do you also multiply the numerator of $\frac{8}{10}$ by 10?*

Listen for You need to multiply both the numerator and denominator of $\frac{8}{10}$ by the same number to find an equivalent fraction.

Generalize *How is finding common denominators useful in solving a problem about comparing fractions with different numerators and different denominators?* Have students explain their reasoning. Listen for understanding that finding an equivalent fraction with the same denominator as another fraction allows you to compare the numerators and tell which fraction is greater or less.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers and that one way to compare fractions is to rewrite one fraction to have the same denominator as the other fraction.
- Explain that on this page students will compare the same two fractions by rewriting one fraction to have the same numerator as the other in order to compare them.

Monitor and Confirm

1 – 5 Check for understanding that:

- the numerator 2 is multiplied by 4 to get a numerator of 8, so multiply the denominator 100 by 4 to get an equivalent fraction of $\frac{8}{400}$
- both area models are the same size
- the model divided into 400 equal parts has smaller parts than the model divided into 10 equal parts
- the model with the greater area shaded represents the greater fraction

Support Whole Class Discussion

6 Be sure students recognize that both methods—finding a common denominator or finding a common numerator—lead to the same solution: $\frac{8}{10} > \frac{2}{100}$

Ask *How does this solution compare to the solution found by using a common denominator? Why do you think this is true?*

Listen for The solutions are the same. In both cases, one fraction is rewritten as an equivalent fraction with either a common numerator or common denominator, so the relationship between the fractions $\frac{8}{10}$ and $\frac{2}{100}$ remains the same.

7 Look for the idea that the fraction with the lesser denominator has equal parts that are larger, so it is the greater fraction.

8 REFLECT

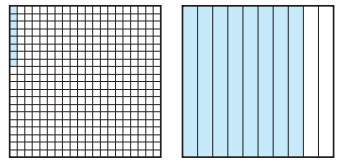
Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to compare fractions by finding a common numerator.

1 What is an equivalent fraction for $\frac{2}{100}$ that has a numerator of 8? $\frac{8}{400}$

2 One model is divided into 400 equal parts, and the other is divided into 10 equal parts. Which model has smaller parts?



the model divided into 400 equal parts

3 Shade 8 parts of each model.

4 Which model has a greater area shaded? **the model divided into 10 equal parts**

5 Which fraction is greater, $\frac{8}{400}$ or $\frac{8}{10}$? $\frac{8}{10}$

6 Which weighs more, the grasshopper or the beetle? **the beetle**

7 Look at the denominators of $\frac{8}{400}$ and $\frac{8}{10}$. When two fractions have the same numerator and different denominators, how do you know which fraction is greater? Explain.

Possible explanation: The fraction with the lesser denominator has bigger parts, so it is greater.

8 REFLECT

Look back at your **Try It**, strategies by classmates, **Model Its**, and the **Connect It** problems on this page. Which models or strategies do you like best for comparing fractions? Explain.

Possible explanation: I like using the shaded models because I can see exactly how much of the model is shaded for each fraction and how big the parts are, so I can easily compare the fractions.

Visual Model

Use drawings of fractions with the same numerator and different denominators to compare fractions.

If . . . *students are having trouble comparing fractions with the same numerators and different denominators that are greater, such as 400,*

Then . . . *use this activity to have them compare visual models of fractions with the same numerators and different denominators that are lesser.*

Use drawings to visually model two fractions with the same numerators and different denominators in order to compare them.

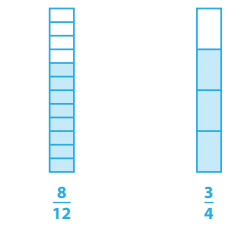
- Draw two same-sized area models to represent $\frac{2}{5}$ and $\frac{2}{10}$ on the board.
- Ask: *What fractions do these area models represent?* [$\frac{2}{5}$ and $\frac{2}{10}$] *Which model has a greater area shaded?* [the model with 2 parts out of 5 shaded]
- Point out that both models have the same number of parts shaded. Ask: *Why is $\frac{2}{5} > \frac{2}{10}$?* [Each fifths part is larger than each tenths part.]
- Repeat with other pairs of fractions that have the same numerators and different denominators, such as $\frac{3}{8}$ and $\frac{3}{4}$, and $\frac{5}{12}$ and $\frac{5}{10}$.

APPLY IT

Use what you just learned to solve these problems.

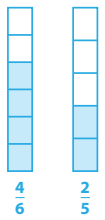
9 Mel's tomato plant is $\frac{8}{12}$ of a foot tall. Her pepper plant is $\frac{3}{4}$ of a foot tall. Compare the heights of the plants using <, >, or =. Use a model to show your comparison. Show your work.

Possible student work:



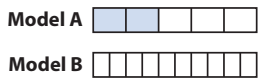
Solution $\frac{8}{12} < \frac{3}{4}$ or $\frac{3}{4} > \frac{8}{12}$

10 Compare the fractions $\frac{4}{6}$ and $\frac{2}{5}$ using <, >, or =. Use a model to show your comparison. Show your work. **Possible student work:**



Solution $\frac{4}{6} > \frac{2}{5}$ or $\frac{2}{5} < \frac{4}{6}$

11 Morgan has the two fraction models shown. Morgan shades Model B to show a fraction less than the fraction shown by Model A. How many parts of Model B could she have shaded? Explain.



Morgan could have shaded 1, 2, or 3 parts of Model B. Possible explanation: Model A represents $\frac{2}{5}$. Model B shows tenths. Since $\frac{2}{5} = \frac{4}{10}$, the number of parts she could have shaded in Model B must be less than 4.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing fractional parts accurately is difficult and here precise measurements are not necessary.

9 $\frac{8}{12} < \frac{3}{4}$ or $\frac{3}{4} > \frac{8}{12}$; See possible work on the Student Worktext page. Students may also use a common denominator to rewrite $\frac{3}{4}$ as $\frac{9}{12}$ and then compare $\frac{9}{12}$ with $\frac{8}{12}$ or use a number line marked in fourths and twelfths to compare the fractions.

10 $\frac{4}{6} > \frac{2}{5}$ or $\frac{2}{5} < \frac{4}{6}$; See possible work on the Student Worktext page. Students may also use a common numerator to rewrite $\frac{2}{5}$ as $\frac{4}{10}$ and then compare $\frac{4}{10}$ with $\frac{4}{6}$.

Close: Exit Ticket

11 Morgan could have shaded 1, 2, or 3 parts of Model B; See possible explanation on the Student Worktext page.

Students' solutions should indicate understanding of:

- how area models represent fractions
- how to represent equivalent fractions with area models
- using area models or equivalent fractions to compare two fractions with different denominators

Error Alert **If** students think that Morgan could have shaded 4 parts of Model B, **then** have students shade 4 parts of the tenths model in Model B and compare the shaded parts of Models A and B to see that both models have the same area shaded. Students can recognize that this indicates that the fractions have the same value and that 4 shaded parts of Model B does not represent a fraction less than the $\frac{2}{5}$ shown in Model A.

Solutions

1. $\frac{3}{4} < \frac{5}{6}$; Students should shade 3 of the 4 parts of the top area model and 5 of the 6 parts of the bottom area model; See shaded area models on the student page.
Basic

2. $\frac{9}{12} < \frac{10}{12}$; Students should divide each equal part in the top area model in problem 1 into 3 equal parts and each equal part in the bottom area model into 2 equal parts for a total of 12 equal parts in both models. See divided area models in problem 1 on the student page.
Medium

3. **a.** $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$
b. $\frac{8}{12} < \frac{9}{12}$. So, $\frac{2}{3} < \frac{9}{12}$.
Medium

Name: _____ LESSON 18 SESSION 2

Practice with Common Numerators and Denominators

Study the Example showing how to compare fractions by finding a common denominator. Then solve problems 1–7.

EXAMPLE

A length of ribbon is $\frac{3}{4}$ of a foot. Another length of ribbon is $\frac{5}{6}$ of a foot.
Compare the lengths using a symbol.

Find a common denominator. $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

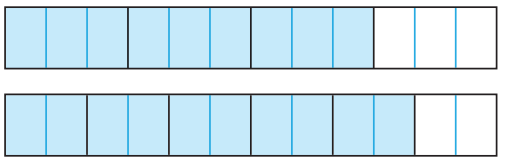
Write the equivalent fractions. $\frac{3}{4} = \frac{9}{12}$ $\frac{5}{6} = \frac{10}{12}$

Compare the numerators. $\frac{9}{12} < \frac{10}{12}$

Since $9 < 10$, that means $\frac{9}{12} < \frac{10}{12}$.

$\frac{3}{4} < \frac{5}{6}$

1. Shade the models to show $\frac{3}{4}$ and $\frac{5}{6}$. Compare the fractions. Write <, >, or =.
$\frac{3}{4}$ (<) $\frac{5}{6}$



2. Divide each model in problem 1 into 12 equal parts to show an equivalent fraction. Write the equivalent fractions and symbol to show the comparison.
$\frac{9}{12} < \frac{10}{12}$

3. Compare $\frac{2}{3}$ and $\frac{9}{12}$ by finding a common denominator.
a. Write a fraction equivalent to $\frac{2}{3}$ with a denominator of 12. $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$
b. Compare the fractions. $\frac{8}{12} < \frac{9}{12}$. So, $\frac{2}{3} < \frac{9}{12}$.

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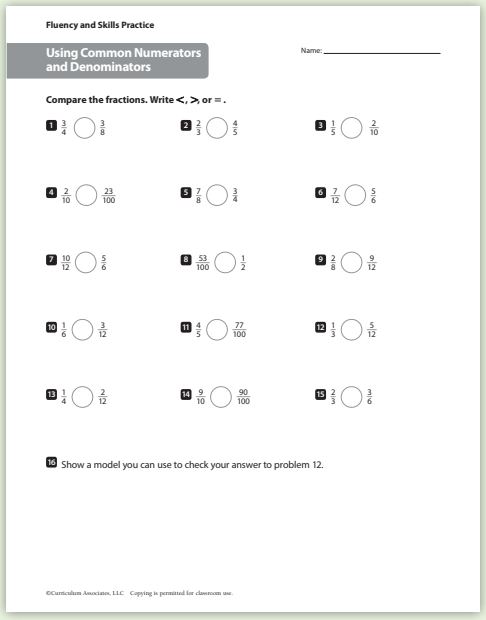
Fluency & Skills Practice

Teacher Toolbox

Assign *Using Common Numerators and Denominators*

In this activity students practice comparing fractions. Students could compare the fractions by drawing a model or finding a common denominator. Students may encounter comparing fractions with different denominators in real-world situations, such as comparing different amounts of the same-size whole (e.g., comparing $\frac{3}{4}$ of a pie to $\frac{5}{8}$ of an equal-sized pie).



Fluency and Skills Practice

Using Common Numerators and Denominators

Name: _____

Compare the fractions. Write <, >, or = .

1. $\frac{3}{4}$ ◯ $\frac{3}{8}$
2. $\frac{2}{3}$ ◯ $\frac{4}{5}$
3. $\frac{1}{5}$ ◯ $\frac{2}{10}$
4. $\frac{2}{10}$ ◯ $\frac{23}{100}$
5. $\frac{7}{8}$ ◯ $\frac{3}{4}$
6. $\frac{7}{12}$ ◯ $\frac{5}{6}$
7. $\frac{10}{12}$ ◯ $\frac{5}{6}$
8. $\frac{53}{100}$ ◯ $\frac{1}{2}$
9. $\frac{2}{8}$ ◯ $\frac{9}{12}$
10. $\frac{1}{6}$ ◯ $\frac{3}{12}$
11. $\frac{4}{5}$ ◯ $\frac{77}{100}$
12. $\frac{1}{3}$ ◯ $\frac{5}{12}$
13. $\frac{1}{4}$ ◯ $\frac{2}{12}$
14. $\frac{9}{10}$ ◯ $\frac{90}{100}$
15. $\frac{2}{3}$ ◯ $\frac{3}{6}$

16. Show a model you can use to check your answer to problem 12.

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4 **a.** $\frac{1 \times 2}{5 \times 2} = \frac{2}{10}$

b. $\frac{2}{10} > \frac{2}{12}$. So, $\frac{1}{5} > \frac{2}{12}$.

Medium

5 **a.** $\frac{2}{5} < \frac{8}{10}$

b. $\frac{5}{12} > \frac{1}{3}$

c. $\frac{3}{5} = \frac{60}{100}$

d. $\frac{9}{100} < \frac{9}{10}$

Medium

6 **B** (**False**);
C (**True**);
E (**True**);
H (**False**);
J (**False**)

Medium

7 No; See possible explanation on the student page.

Challenge

4 Compare $\frac{1}{5}$ and $\frac{2}{12}$ by finding a common numerator.

a. Write a fraction equivalent to $\frac{1}{5}$ with a numerator of 2. $\frac{1 \times \boxed{2}}{5 \times \boxed{2}} = \frac{2}{\boxed{10}}$

b. Compare the fractions. $\frac{2}{10}$ (>) $\frac{2}{12}$. So, $\frac{1}{5}$ (>) $\frac{2}{12}$.

5 Compare the fractions. Use the symbols <, >, and =.

a. $\frac{2}{5}$ (<) $\frac{8}{10}$

b. $\frac{5}{12}$ (>) $\frac{1}{3}$

c. $\frac{3}{5}$ (=) $\frac{60}{100}$

d. $\frac{9}{100}$ (<) $\frac{9}{10}$

6 Tell whether each comparison is *True* or *False*.

	True	False
$\frac{2}{3} > \frac{5}{6}$	Ⓐ	Ⓑ
$\frac{4}{10} < \frac{4}{5}$	Ⓒ	Ⓓ
$\frac{70}{100} = \frac{7}{10}$	Ⓔ	Ⓕ
$\frac{1}{3} > \frac{3}{1}$	Ⓖ	Ⓗ
$\frac{3}{4} < \frac{2}{3}$	Ⓘ	Ⓙ

7 Can two fractions with the same numerator and different denominators be equal? Use words and numbers to explain.

No. Possible explanation: Fractions with the same numerator have the same number of parts, but the size of the parts is different when the denominators are different. The fractions can't be equal, because the fraction with the smaller-size parts is the smaller fraction. For example: $\frac{3}{4}$ is greater than $\frac{3}{5}$ because fourths are greater than fifths, $\frac{3}{4} > \frac{3}{5}$.

Vocabulary

common denominator a number that is a common multiple of the denominators of two or more fractions.

denominator the number below the line in a fraction that tells the number of equal parts in the whole.

numerator the number above the line in a fraction that tells the number of equal parts that are being described.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Use with ***Apply It*** problem 7. Have students listen as you think through using a benchmark fraction to find the greater fraction. Draw a number line to use as you model the process.

- *Divide the number line into 8 equal sections.*
- *Find and label the benchmark fraction.* [$\frac{1}{2}$]
- *Find and label* $\frac{4}{8}$. [Emphasize that $\frac{4}{8}$ is located at the same position as $\frac{1}{2}$.]
- *Find and label* $\frac{3}{4}$.
- $\frac{3}{4}$ *is greater than* $\frac{4}{8}$.

Ask students to explain how to use a benchmark fraction to find the greater fraction in their own words.

Levels 2–4

Listening/Speaking Use with ***Apply It*** problem 7. Say: *Draw a number line divided into 8 equal sections*. Ask students to form pairs. Have them listen to, discuss, and respond to the following questions as they think through the process to find the greater fraction.

- *What fraction will you use as the benchmark fraction?*
- *Where will you label* $\frac{4}{8}$*? What connection do you see between* $\frac{4}{8}$ *and* $\frac{1}{2}$*?*
- *Where will you label* $\frac{3}{4}$*?*
- *Which fraction is greater? How do you know?*

Call on pairs to share their responses.

Levels 3–5

Reading Use with ***Apply It*** problem 7. Say: *Draw a number line to use as you think through the process to find the greater fraction*. Write the following questions and have students read them before solving the problem.

- *How many sections will you divide the number line into?*
- *What fraction will you use as the benchmark fraction?*
- *Where will you label* $\frac{4}{8}$*? What is the connection between* $\frac{4}{8}$ *and* $\frac{1}{2}$*?*
- *Where will you label* $\frac{3}{4}$*?*
- *Which fraction is greater? How do you know?*

Call on students to share their results.

LESSON 18

SESSION 3 Develop

Purpose In this session, students solve a problem that requires comparing two fractions with different numerators and different denominators. Students model the fractions in the problem either on paper or with manipulatives to determine the greater fraction. The purpose is to have students develop strategies for comparing fractions with different numerators and different denominators.

Start

Connect to Prior Knowledge

Why Review comparing fractions with the same denominators on a number line to prepare students to compare fractions with different numerators and denominators using a benchmark.

How Use a number line to compare fractions that have different numerators and the same denominator.

Tell whether each comparison is *True* or *False*.

❶ $\frac{1}{6} > \frac{5}{6}$ ❷ $\frac{2}{6} < \frac{4}{6}$

Solutions

1. False
2. True

Develop Language

Why Practice reading fractions that have the same numerator and denominator.

How Remind students that when they see a fraction with the same numerator and denominator, the fraction names 1 whole. Write several fractions with the same numerators and denominators. Model how to read them. Tell students that the fractions name 1 whole. For example, they can read the fraction $\frac{10}{10}$ as *ten tenths*.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the fractions and the unit of measurement.

Ask *What fraction tells the amount of time for the swimming lesson? the amount of time for homework? What unit of measurement for time is given?*

Develop Using a Benchmark to Compare Fractions

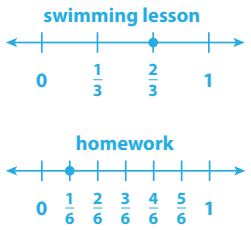
Read and try to solve the problem below.

Jasmine's swimming lesson lasts for $\frac{2}{3}$ of an hour. It takes her $\frac{1}{6}$ of an hour to do her homework. Does Jasmine spend more time on her homework or at her swimming lesson?

TRY IT

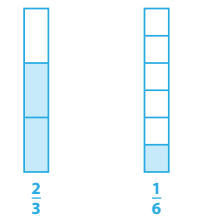
Possible student work:

Sample A



Jasmine spends more time at her swimming lesson than on her homework.

Sample B



She spends more time at her swimming lesson than she does on homework.

Math Toolkit

- fraction circles
- fraction tiles
- number lines
- fraction bars
- index cards
- fraction models

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: I knew . . . so I . . .

DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *thirds* and *sixths* as they discuss their solutions.

Support as needed with questions such as:

- *Have you solved a problem like this before?*
- *How is your solution method the same as or different from your partner's method?*

Common Misconception Look for students who find the correct result but reason incorrectly, thinking that $\frac{2}{3} > \frac{1}{6}$ because $2 > 1$ without taking into account the different denominators. Have students use fraction tiles to explain the comparison.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction circles or fraction tiles modeling two thirds and one sixth
- area models or labeled number lines representing two thirds and one sixth
- writing equivalent fractions to compare the two fractions
- reasoning using a benchmark fraction of $\frac{1}{2}$ to compare the two fractions

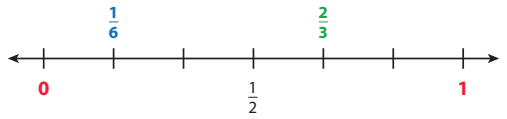
Explore different ways to understand using benchmarks to compare fractions.

Jasmine's swimming lesson lasts for $\frac{2}{3}$ of an hour. It takes her $\frac{1}{6}$ of an hour to do her homework. Does Jasmine spend more time on her homework or at her swimming lesson?

MODEL IT

You can use a number line to help you compare fractions.

The number line shows where the fractions $\frac{2}{3}$ and $\frac{1}{6}$ are compared to 0 and 1.



The number line shows that $\frac{1}{6}$ is closer to 0 than $\frac{2}{3}$ is.
It also shows that $\frac{2}{3}$ is closer to 1 than $\frac{1}{6}$ is.
This means that $\frac{1}{6} < \frac{2}{3}$ and $\frac{2}{3} > \frac{1}{6}$.

SOLVE IT

You can use a benchmark fraction to solve the problem.

Another way to compare fractions is by using a benchmark fraction.
Use $\frac{1}{2}$ as a benchmark to compare $\frac{1}{6}$ and $\frac{2}{3}$.

The number line shows that $\frac{1}{6}$ is less than $\frac{1}{2}$ and $\frac{2}{3}$ is greater than $\frac{1}{2}$.
So, $\frac{1}{6} < \frac{2}{3}$ and $\frac{2}{3} > \frac{1}{6}$.

Jasmine spends more time at her swimming lesson than on homework.

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show $\frac{2}{3}$? Where does your model show $\frac{1}{6}$? How does your model show that the thirds and sixths represent the same-sized whole?*

Listen for Students should recognize that accurate responses include area models divided into 3 equal parts with 2 parts shaded and into 6 equal parts with 1 part shaded and that both models are the same size. Responses may also include labeled thirds and sixths number lines showing the same distance between 0 and 1.

MODEL IT & SOLVE IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of thirds representing part of an hour
- the number of sixths representing part of an hour

Ask *How does each model represent the thirds? How does each model represent the sixths?*

Listen for Both number lines go from 0 to 1 and are divided into sixths, so they also show thirds.

For a number line, prompt students to identify how the comparison of the fractions is shown.

- *What does the fraction $\frac{1}{6}$ represent?*
- *How is the amount of time Jasmine swims represented?*
- *How does the number line help you compare fractions?*

For a number line with a benchmark fraction, prompt students to identify how to use a benchmark fraction to solve the problem.

- *How is this number line model the same as the other number line model? How is it different?*
- *What is the purpose of showing $\frac{1}{2}$ on the number line?*
- *How can a benchmark fraction help you compare two other fractions?*

Deepen Understanding

Benchmark Fractions

SMP 4 Model with mathematics.

When discussing the model of a number line with a benchmark fraction, tell students that a *benchmark fraction* is a common fraction, such as $\frac{1}{2}$ or $\frac{3}{4}$, that you can compare other fractions to.

Ask *What does the location of $\frac{1}{2}$ on the number line compared to $\frac{1}{6}$ and to $\frac{2}{3}$ tell you about how to choose a benchmark fraction?*

Listen for When comparing fractions, it is best to use a benchmark fraction that is greater than one of the fractions and less than the other fraction.

Ask *Whole numbers can also be used as benchmarks. Would 1 be as useful as $\frac{1}{2}$ to use as a benchmark to compare the fractions $\frac{1}{6}$ and $\frac{2}{3}$? Explain.*

Listen for No, 1 would not be as useful because 1 is greater than both fractions. You would know that both fractions are less than 1, but not how the fractions compare to each other.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers and that one way to compare two fractions with different numerators and denominators is to use a benchmark fraction such as $\frac{1}{2}$.
- Explain that on this page students will compare two different fractions by using another benchmark instead of $\frac{1}{2}$.

Monitor and Confirm

1–4 Check for understanding that:

- $\frac{10}{10}$ is equal to 1
- $\frac{11}{10}$ is greater than 1 and $\frac{7}{8}$ is less than 1
- using a benchmark of 1 helps determine that $\frac{11}{10}$ is greater than $\frac{7}{8}$
- the symbol $>$ means "is greater than"

Support Whole Class Discussion

1–4 Tell students that these problems will prepare them to provide the explanation required in problem 5.

Be sure students understand that $\frac{1}{2}$ is not the only benchmark you can use to compare two fractions.

Ask *Why do you think that $\frac{1}{2}$ is not used as a benchmark to compare the fractions $\frac{11}{10}$ and $\frac{7}{8}$?*
Listen for The fractions $\frac{11}{10}$ and $\frac{7}{8}$ are both greater than $\frac{1}{2}$, so comparing them to $\frac{1}{2}$ does not help you tell which fraction is greater or less than the other.

Ask *Why do you think 1 is used as the benchmark to compare $\frac{11}{10}$ and $\frac{7}{8}$?*
Listen for The fraction $\frac{7}{8}$ is less than 1 and the fraction $\frac{11}{10}$ is greater than 1.

5 Look for the idea that you can use a benchmark to compare two fractions with different numerators and different denominators. This is useful especially if one fraction is greater than the benchmark and the other fraction is less than the benchmark.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will solve a similar problem using 1 as a benchmark. Think about the two fractions $\frac{11}{10}$ and $\frac{7}{8}$.

1 Which fraction, $\frac{11}{10}$ or $\frac{7}{8}$, is greater than 1? $\frac{11}{10}$

2 Which fraction, $\frac{11}{10}$ or $\frac{7}{8}$, is less than 1? $\frac{7}{8}$

3 Which fraction, $\frac{11}{10}$ or $\frac{7}{8}$, is greater? Explain.

$\frac{11}{10}$ is greater. Possible explanation: Since $\frac{11}{10}$ is greater than 1 and $\frac{7}{8}$ is less than 1, $\frac{11}{10}$ must be greater than $\frac{7}{8}$.

4 Write $<$, $>$, or $=$ to show the comparison. $\frac{11}{10}$ $>$ $\frac{7}{8}$

5 Explain how you can use benchmarks to compare fractions.

You can compare both fractions to the same number to see which fraction is greater than, less than, or equal to that benchmark. The fraction that is greater than the benchmark is greater than the fraction that is less than or equal to the benchmark.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model It** and **Solve It**. Which models or strategies do you like best for using benchmarks to compare fractions? Explain.

Possible explanation: I like the strategy of using a number line best because I can see that $\frac{2}{3}$ is closer to 1 than $\frac{1}{6}$ so $\frac{2}{3}$ must be greater than $\frac{1}{6}$.

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Hands-On Activity

Use a number line and fraction cards to compare fractions.

If . . . *students are unsure about using a benchmark fraction to compare two fractions,*

Then . . . *use a number line and fraction cards to provide a concrete model to connect to the visual and symbolic models.*

Materials For each pair: 10 index cards labeled with the fractions $\frac{1}{3}$, $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{4}{10}$, $\frac{3}{12}$, Activity Sheet *Number Lines*

- Have students compare two fractions using the benchmark fraction, $\frac{1}{2}$.
- Have students label 0, 1, and $\frac{1}{2}$ on a number line.
- Give partners a set of fraction cards. Have one partner choose two fractions to place on the number line, one between 0 and $\frac{1}{2}$ and one between $\frac{1}{2}$ and 1. Have students explain each placement and discuss with their partners any fractions whose locations they are not sure about.
- Have the partners write a comparison statement comparing the two fractions and justify the statement in terms of $\frac{1}{2}$.
- Have partners repeat with other pairs of fractions.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing equal intervals on number lines is challenging and here exact spacing between marks on a number line is not necessary.

7 $\frac{4}{8} = \frac{1}{2}$ and $\frac{3}{4} > \frac{1}{2}$. So $\frac{3}{4} > \frac{4}{8}$. See possible work on the Student Worktext page.

8 Nathan walks a greater distance than Sarah; Students should use the benchmark number 1 to compare the fractions $\frac{10}{10}$ and $\frac{11}{12}$. See possible explanation on the Student Worktext page.

Close: Exit Ticket

9 **B;** Students may first compare $\frac{4}{6}$ to $\frac{1}{2}$ by using a common denominator and reasoning that $\frac{1}{2} = \frac{3}{6}$ and $\frac{4}{6} > \frac{3}{6}$, so $\frac{4}{6} > \frac{1}{2}$. Students may then compare $\frac{3}{8}$ to $\frac{1}{2}$ by using a common denominator and reasoning that $\frac{1}{2} = \frac{4}{8}$ and $\frac{3}{8} < \frac{4}{8}$, so $\frac{3}{8} < \frac{1}{2}$. Because $\frac{4}{6} > \frac{1}{2}$ and $\frac{3}{8} < \frac{1}{2}$, $\frac{4}{6} > \frac{3}{8}$.

Error Alert **If** students choose A or C and incorrectly compare the fractions, **then** review how to use the benchmark fraction $\frac{1}{2}$ to compare fractions by drawing a sixths number line labeled with $\frac{1}{2}$ and an eighths number line labeled with $\frac{1}{2}$, locating $\frac{4}{6}$ and $\frac{3}{8}$ on their respective number lines and comparing the locations of both fractions to $\frac{1}{2}$.

APPLY IT

Use what you just learned to solve these problems.

7 Tell which fraction is greater, $\frac{4}{8}$ or $\frac{3}{4}$. Use the benchmark fraction $\frac{1}{2}$ to explain your answer. Show your work.

Solution $\frac{3}{4} > \frac{4}{8}$; Possible explanation: $\frac{4}{8} = \frac{1}{2}$ and $\frac{3}{4} > \frac{1}{2}$. So, $\frac{3}{4} > \frac{4}{8}$.

8 Nathan walks $\frac{10}{10}$ of a mile. Sarah walks $\frac{11}{12}$ of a mile. Who walks a greater distance? Explain. Use a benchmark number in your explanation.

Solution Nathan walks a greater distance than Sarah.
Possible explanation: 10 out of 10 is equal to 1. $\frac{11}{12}$ is less than 1. So, $\frac{10}{10} > \frac{11}{12}$.

9 Use the benchmark fraction $\frac{1}{2}$ to compare the two fractions below. Which symbol correctly compares the fractions?

$\frac{4}{6}$ ◯ $\frac{3}{8}$

Ⓐ <
Ⓑ >
Ⓒ =
Ⓓ +

Solutions

1 **a.** See labeled number line on the student page.

b. $\frac{3}{2}$

c. $\frac{9}{10}$

d. $\frac{9}{10} < \frac{3}{2}$; Students' explanations should include using the benchmark number 1 to compare the fractions. See possible explanation on the student page.

Medium

Name: _____ **LESSON 18 SESSION 3**

Practice Using a Benchmark to Compare Fractions

Study the Example showing how to use 1 as a benchmark to compare fractions. Then solve problems 1–4.

EXAMPLE

Carol compares $\frac{3}{4}$ and $\frac{2}{1}$. She says $\frac{3}{4} > \frac{2}{1}$ because both the numerator and the denominator in $\frac{3}{4}$ are greater than the numerator and denominator in $\frac{2}{1}$. $3 > 2$ and $4 > 1$. Is Carol correct?

Compare each fraction to the benchmark 1.

$\frac{3}{4} < 1$ and $\frac{2}{1} > 1$

$\frac{3}{4} < \frac{2}{1}$ and $\frac{2}{1} > \frac{3}{4}$. Carol is not correct.

1 Compare $\frac{9}{10}$ and $\frac{3}{2}$.

a. Label $\frac{9}{10}$ and $\frac{3}{2}$ on the number line below.

b. Which fraction is greater than 1? $\frac{3}{2}$

c. Which fraction is less than 1? $\frac{9}{10}$

d. Write <, >, or = to show the comparison. Explain how you found your answer. $\frac{9}{10} < \frac{3}{2}$

Possible explanation: $\frac{9}{10}$ is less than $\frac{3}{2}$ because $\frac{9}{10}$ is less than 1 and $\frac{3}{2}$ is greater than 1.

Vocabulary

benchmark fraction a common fraction that you might compare other fractions to. For example, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are often used as benchmark fractions.

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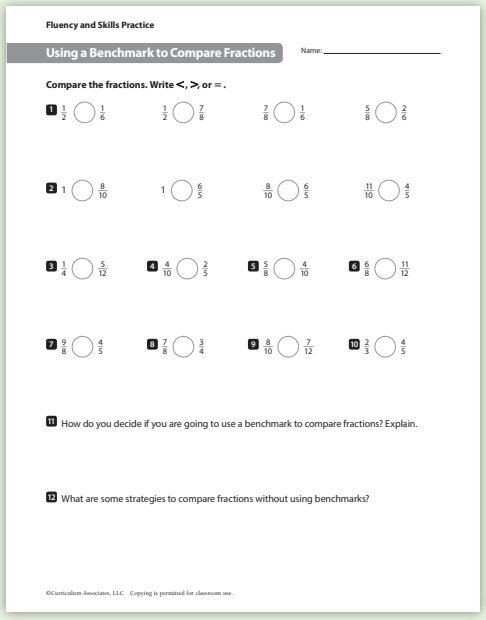
Fluency & Skills Practice Teacher Toolbox

Assign *Using a Benchmark to Compare Fractions*

In this activity students practice comparing fractions. Students may use a benchmark, such as $\frac{1}{2}$ or 1, to compare the fractions, or they may use another strategy. It is useful for students to consider different strategies, as one strategy may be easier or more efficient to use in a real-world situation.



Fluency and Skills Practice

Using a Benchmark to Compare Fractions Name: _____

Compare the fractions. Write <, >, or = .

1 $\frac{1}{2}$ ◯ $\frac{1}{6}$ $\frac{1}{2}$ ◯ $\frac{7}{8}$ $\frac{7}{8}$ ◯ $\frac{1}{6}$ $\frac{5}{8}$ ◯ $\frac{2}{6}$

2 1 ◯ $\frac{8}{10}$ 1 ◯ $\frac{6}{5}$ $\frac{8}{10}$ ◯ $\frac{6}{5}$ $\frac{11}{10}$ ◯ $\frac{4}{5}$

3 $\frac{1}{4}$ ◯ $\frac{5}{12}$ 4 $\frac{4}{10}$ ◯ $\frac{2}{5}$ 5 $\frac{5}{8}$ ◯ $\frac{4}{10}$ 6 $\frac{6}{8}$ ◯ $\frac{11}{12}$

7 $\frac{9}{8}$ ◯ $\frac{4}{5}$ 8 $\frac{7}{8}$ ◯ $\frac{3}{4}$ 9 $\frac{8}{10}$ ◯ $\frac{7}{12}$ 10 $\frac{2}{3}$ ◯ $\frac{4}{5}$

11 How do you decide if you are going to use a benchmark to compare fractions? Explain.

12 What are some strategies to compare fractions without using benchmarks?

2 **a.** See labeled number line on the student page.

b. $\frac{5}{6}$

c. $\frac{1}{3}$

d. $\frac{5}{6} > \frac{1}{3}$; Students' explanations should include using the benchmark fraction $\frac{1}{2}$ to compare the fractions. See possible explanation on the student page.

Medium

3 $\frac{7}{10} > \frac{5}{12}$; Students' explanations should include using the benchmark fraction $\frac{1}{2}$ to compare the fractions. See possible explanation on student page.

Medium

4 **True**; **1**;

True; $\frac{1}{2}$;

False; $\frac{1}{2}$;

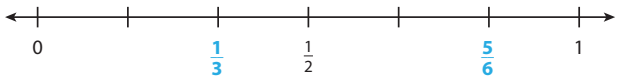
False; **1**;

False; **1**

Challenge

2 Compare $\frac{5}{6}$ and $\frac{1}{3}$ using the benchmark fraction $\frac{1}{2}$.

a. Label $\frac{5}{6}$ and $\frac{1}{3}$ on the number line below.



b. Which fraction is greater than $\frac{1}{2}$? $\frac{5}{6}$

c. Which fraction is less than $\frac{1}{2}$? $\frac{1}{3}$

d. Write <, >, or = to show the comparison. Explain how you found your answer.

$\frac{5}{6} > \frac{1}{3}$

Possible explanation: $\frac{5}{6}$ is greater than $\frac{1}{3}$ because $\frac{5}{6}$ is greater than $\frac{1}{2}$ and $\frac{1}{3}$ is less than $\frac{1}{2}$.

3 Use a benchmark fraction to compare the fractions $\frac{7}{10}$ and $\frac{5}{12}$. Explain how you found your answer.

$\frac{7}{10} > \frac{5}{12}$; Possible explanation: $\frac{1}{2}$ is equal to $\frac{5}{10}$, so $\frac{7}{10}$ is greater than $\frac{1}{2}$. $\frac{1}{2}$ is also equal to $\frac{6}{12}$, so $\frac{5}{12}$ is less than $\frac{1}{2}$. $\frac{7}{10}$ is greater than $\frac{5}{12}$.

4 Write *True* or *False* for each comparison. Then write the benchmark you could use to compare the fractions.

	True* or *False	**Benchmark**
$\frac{9}{8} > \frac{11}{12}$	True	1
$\frac{2}{5} < \frac{5}{6}$	True	$\frac{1}{2}$
$\frac{7}{10} < \frac{2}{4}$	False	$\frac{1}{2}$
$\frac{4}{5} > \frac{2}{2}$	False	1
$\frac{3}{2} < \frac{9}{10}$	False	1

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Writing Read ***Apply It*** problem 1 to students. Write *Use a Number Line, Find a Common Numerator,* and *Find a Common Denominator*. Remind students they can use these strategies to determine which fraction is greater and to find who finishes more homework. Put students into groups of 3. Ask Student A to solve the problem using *Use a Number Line,* Student B to solve it using *Find a Common Numerator,* and Student C to solve it using *Find a Common Denominator*. When students have completed the problem, ask them to compare their results. Ask: *Who finishes more problems? How do you know?* Have students write their responses using the sentence frame: _____ *finishes more problems because* _____ *is greater than* _____.

Levels 2–4

Speaking/Writing Choral read ***Apply It*** problem 1. Ask students to think of strategies they could use to solve the problem. Answers may include *use a number line, find a common numerator,* and *find a common denominator*. Divide students into three groups. Assign each group a strategy and then have them solve the problem. As they solve the problems, encourage students to discuss the process. Have groups compare their results. Ask: *Who finishes more homework problems? How do you know?* Have students write their responses using the following sentence frame: *I determined* _____ *finishes more homework problems because* _____.

Levels 3–5

Listening/Speaking Have students form pairs and read ***Apply It*** problem 1. Ask them to make a list of strategies they could use to solve the problem. Assign each pair a strategy. When they have solved the problem, ask pairs to compare their results. Ask: *How did you solve the problem using the strategy you were assigned?* Encourage students to ask questions as they listen to other students describe the process.

- *Why did you decide to use* _____*?*
- *What did you think of first?*
- *Did you make a prediction about which fraction was greater before you solved the problem?*

LESSON 18

SESSION 4 Refine

Purpose In this session, students solve word problems involving comparing fractions with different numerators and denominators and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: Activity Sheet *Number Lines*

Why Confirm understanding of comparing fractions.

How Have students compare $\frac{7}{10}$ and $\frac{4}{5}$ and use an area model or number line to explain the comparison.

Compare $\frac{7}{10}$ and $\frac{4}{5}$.

Use an area model or number line to explain the comparison.

Solution

$\frac{7}{10} < \frac{4}{5}$.

Students' models should show 10 and 5 equal parts, representing $\frac{7}{10}$ as less than $\frac{4}{5}$.

LESSON 18 SESSION 4 ● ● ● ●

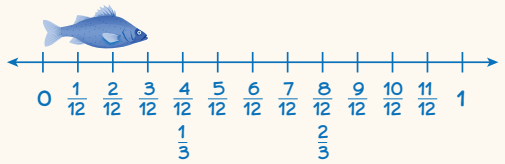
Refine Comparing Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Becker catches a fish that is $\frac{3}{12}$ of a yard long. The fish has to be longer than $\frac{1}{3}$ of a yard in order to keep it. Can Becker keep the fish?

Look at how you could show your work using a number line.



Solution **Since $\frac{3}{12}$ is less than $\frac{1}{3}$, Becker can't keep the fish.**

It is important that both measurements use the same unit!

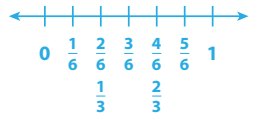
PAIR/SHARE
How else could you solve this problem?

APPLY IT

1 Myron and Jane work on the same set of homework problems. Myron finishes $\frac{5}{6}$ of the problems, and Jane finishes $\frac{2}{3}$ of the problems. Who finishes more of their homework problems? Show your work.

Which strategy for comparing do you think works best with these fractions?

Possible student work:



$\frac{2}{3} = \frac{4}{6}$

Since $\frac{5}{6} > \frac{4}{6}$, $\frac{5}{6} > \frac{2}{3}$.

Solution **Myron finishes more homework problems.**

PAIR/SHARE
How did you and your partner decide what strategy to use to solve the problem?

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Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
$\frac{7}{10}$ is greater than $\frac{4}{5}$ because $7 > 4$	not understand that denominators must be the same in order to compare numerators.	Have students draw same-sized area models of $\frac{7}{10}$ and $\frac{4}{5}$. Point out that the size of the parts is not the same. Have students divide the model for $\frac{4}{5}$ into 10 equal parts to show an equivalent fraction with a denominator of 10. Have students write the equivalent fraction $\left[\frac{8}{10}\right]$ and then compare it to $\frac{7}{10}$.
$\frac{7}{10} = \frac{4}{5}$ because $\frac{7}{10} > \frac{1}{2}$ and $\frac{4}{5} > \frac{1}{2}$	not understand when it is appropriate to use a benchmark fraction.	Explain that when both fractions are greater than (or less than) a benchmark fraction, there is not enough information to compare. Have students label a number line with 0 and 1. Help them mark and label tenths and fifths. Have students locate $\frac{7}{10}$ and $\frac{4}{5}$ on the number line to make the comparison.

EXAMPLE

Becker cannot keep his fish because $\frac{3}{12}$ of a yard is less than $\frac{1}{3}$ of a yard; a number line showing twelfths and thirds is one way to solve the problem. Students could also solve the problem by using the common denominator 12, writing $\frac{1}{3}$ as $\frac{4}{12}$, and comparing $\frac{3}{12}$ to $\frac{4}{12}$.

Look for The twelfths and thirds must represent the same-sized whole, indicated by both measurements being described as parts of one yard.

APPLY IT

1. Myron finishes more homework problems; Students may use a number line to show equivalent thirds and sixths fractions to compare $\frac{5}{6}$ and $\frac{2}{3}$. Students could also solve the problem by multiplying both the numerator and denominator of $\frac{2}{3}$ by 2 so both fractions have a common denominator of 6.
$\frac{5}{6} > \frac{4}{6}$, so $\frac{5}{6} > \frac{2}{3}$.
DOK 2

   Look for Since 6 is a multiple of 3, students may find it more efficient to use a common denominator to compare the fractions.

2. $\frac{3}{10} < \frac{7}{12}$ or $\frac{7}{12} > \frac{3}{10}$; Students should solve the problem by comparing each fraction to $\frac{1}{2}$.
$\frac{3}{10} < \frac{1}{2}$ and $\frac{7}{12} > \frac{1}{2}$.
DOK 1

   Look for The fraction $\frac{3}{10}$ is less than $\frac{1}{2}$, or $\frac{5}{10}$, and the fraction $\frac{7}{12}$ is greater than $\frac{1}{2}$, or $\frac{6}{12}$.

3. **A;** Use common numerators and then look at the denominators to compare.

   Explain why the other two answer choices are not correct:

   C is not correct because fifths are greater than sixths.

   D is not correct because $\frac{3}{6} = \frac{1}{2}$.
DOK 3

2. Compare the fractions $\frac{3}{10}$ and $\frac{7}{12}$ using the benchmark fraction $\frac{1}{2}$. Show your work.
Possible student work:
$\frac{3}{10} < \frac{1}{2}$
$\frac{7}{12} > \frac{1}{2}$
So, $\frac{3}{10} < \frac{7}{12}$.

Solution $\frac{3}{10} < \frac{7}{12}$ **or** $\frac{7}{12} > \frac{3}{10}$

You already know about how big $\frac{1}{2}$ is!

PAIR/SHARE
Draw a model to check your answer.

There are several ways to compare fractions!

3. Janelle walks $\frac{3}{6}$ of a mile. Pedro walks $\frac{6}{10}$ of a mile. Which statement shows how to find the greater fraction?

   Ⓐ $\frac{3}{6} = \frac{6}{12}$ and $\frac{6}{12} < \frac{6}{10}$

   Ⓑ $\frac{3}{6} = \frac{6}{12}$ and $\frac{6}{12} > \frac{6}{10}$

   Ⓒ $\frac{6}{10} = \frac{3}{5}$ and $\frac{3}{5} < \frac{3}{6}$

   Ⓓ $\frac{3}{6} < \frac{1}{2}$ and $\frac{6}{10} > \frac{1}{2}$

   Tina chose Ⓑ as the correct answer. How did she get that answer?

   Tina found a fraction equivalent to $\frac{3}{6}$ with a numerator of 6, but compared $\frac{6}{12}$ and $\frac{6}{10}$ incorrectly. She thought that $\frac{6}{12}$ is greater than $\frac{6}{10}$ because 12 is greater than 10.

PAIR/SHARE
How can you find the answer using a benchmark fraction?

4 **A**; Find a common denominator: 12. Multiply the numerator and denominator of $\frac{2}{3}$ by 4 and the numerator and denominator of $\frac{3}{4}$ by 3. Then compare the numerators.
DOK 1

Error Alert Students may choose B, C, or D, thinking that they can use a common numerator to compare, without recognizing that the equivalent fraction shown for $\frac{3}{4}$ is incorrect.

5 **A** (>);
E (<);
G (>);
K (<);
O (=)
DOK 2

6 Sam does not practice long enough; Students may compare the fractions using the benchmark fraction $\frac{1}{2}$. $\frac{5}{10} = \frac{1}{2}$ and $\frac{2}{6} < \frac{1}{2}$. So, $\frac{2}{6} < \frac{5}{10}$.
DOK 2

4 Grant uses $\frac{2}{3}$ of a cup of raisins and $\frac{3}{4}$ of a cup of almonds to make trail mix. Which statement can be used to find out if there are more raisins or almonds in the trail mix?

Ⓐ $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$
Ⓑ $\frac{2}{3} = \frac{4}{6}$ and $\frac{3}{4} = \frac{4}{5}$
Ⓒ $\frac{2}{3} = \frac{6}{9}$ and $\frac{3}{4} = \frac{6}{12}$
Ⓓ $\frac{2}{3} = \frac{6}{9}$ and $\frac{3}{4} = \frac{6}{7}$

5 Select >, <, or = to complete a true comparison for each pair of fractions.

	>	<	=
$\frac{8}{3} \square \frac{9}{4}$	Ⓐ	Ⓑ	Ⓒ
$\frac{7}{10} \square \frac{7}{8}$	Ⓓ	Ⓔ	Ⓕ
$\frac{1}{2} \square \frac{3}{8}$	Ⓖ	Ⓗ	Ⓘ
$\frac{2}{4} \square \frac{4}{6}$	Ⓙ	Ⓚ	Ⓛ
$\frac{7}{5} \square \frac{140}{100}$	Ⓜ	Ⓝ	Ⓞ

6 Sam's music teacher tells him to practice his trombone for $\frac{5}{10}$ of an hour. Sam practices for $\frac{2}{6}$ of an hour. Does he practice long enough? Show your work.

Possible student work: $\frac{2}{6} < \frac{1}{2}$ **and** $\frac{5}{10} = \frac{1}{2}$, **so** $\frac{2}{6} < \frac{5}{10}$.

Sam **does not** practice long enough.

Differentiated Instruction

RETEACH

Hands-On Activity
Draw and cut out grid models to compare fractions.

Students struggling with comparing fractions

Will benefit from additional work with concrete representations of fractions

Materials For each pair: scissors, colored pencils, Activity Sheet *1-Centimeter Grid Paper*

- Tell each student to work with a partner to draw and cut out two 3-by-4 arrays. Explain that the arrays show the same wholes and can be used to show halves, thirds, fourths, and twelfths. Have students color part of each of their two array models and write a fraction to show each colored part. Then have students compare the fractions using >, <, or =.
- Next, have students draw and cut out two 2-by-8 arrays. Explain that they can use these two same-sized arrays to model and compare halves, fourths, and eighths fractions. Have students color part of each array, write the fraction each represents, and compare the fractions using >, <, or =.
- Ask students if they can use the 3-by-4 and 2-by-8 array models to compare eighths and twelfths. [No, because the wholes are different sizes.]

EXTEND

Challenge Activity
Compare three or more fractions.

Students who have achieved proficiency

Will benefit from deepening understanding of comparing fractions

Materials For each pair: index cards labeled with halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths fractions

- Give each pair a set of fraction cards. Have partners set out three, four, or five fractions and order them from least to greatest.
- The strategy is to compare one fraction to another, then choose a third fraction, comparing it to each to place it correctly.

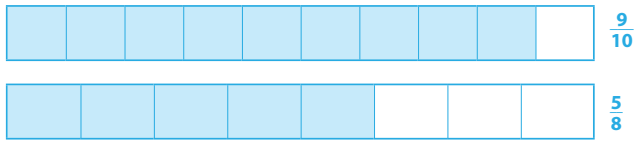
7 $\frac{5}{10} < \frac{5}{8}$; 5 of 10 equal parts is a lesser amount than 5 of 8 equal parts.
DOK 2

8 Rachel sells a greater fraction of her boxes of fruit; Students may draw and shade area models to compare $\frac{9}{10}$ and $\frac{5}{8}$. See possible work on the Student Worktext page.
DOK 2

7 Compare the fractions $\frac{5}{10}$ and $\frac{5}{8}$. Write the symbol >, <, or =.

$\frac{5}{10}$ < $\frac{5}{8}$

8 Rachel and Sierra have the same number of boxes of fruit to sell for a fundraiser. Each box is the same size. Rachel sells $\frac{9}{10}$ of her boxes, and Sierra sells $\frac{5}{8}$ of her boxes. Which girl sells a greater fraction of her boxes of fruit? Draw a model to show your answer. Show your work. **Possible student work:**



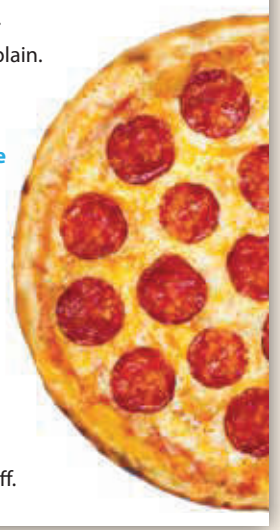
$\frac{9}{10}$ has a greater area shaded, so it is greater than $\frac{5}{8}$.

Rachel sells a greater fraction of her boxes of fruit.

9 MATH JOURNAL

Jeff says $\frac{3}{4}$ of a small pizza is more than $\frac{1}{3}$ of a large pizza. Alicia disagrees. Who is right? Do you have enough information to know who is right? Explain.

You don't have enough information to know who is right. Possible explanation: You need to see the size of the pizzas. If the small pizza is close in size to the large pizza, Jeff could be right. If the large pizza is a lot larger than the small pizza, Alicia could be right.



SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–9

Compare fractions.

All students will benefit from additional work with comparing fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding that the size of the wholes must be the same in order to compare two fractions and that the sizes of the pizzas differ, so there is not enough information to compare $\frac{3}{4}$ of a small pizza to $\frac{1}{3}$ of a large pizza.

Error Alert **If** students do not recognize that the size of the wholes must be the same to compare the fractions $\frac{3}{4}$ and $\frac{1}{3}$ and incorrectly think that Jeff is right, **then** have them draw a small circle divided into fourths with 3 parts shaded and a larger circle divided into thirds with 1 part shaded so they can see that the sizes of the wholes are not the same.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.

LESSON 18

Lesson Quiz

Teacher Toolbox

Tested Skills

Assesses 4.NF.A.2

Problems on this assessment form require students to be able to compare fractions with different numerators and denominators by finding equivalent fractions or using benchmark fractions, and to write fraction comparison statements using the symbols $<$, $=$, and $>$. Students will also need to be familiar with representing fractions with number lines.

Alternately, teachers may assign the **Digital Comprehension Check** online to assess student understanding of this material.

Error Alert Students may:

- compare only the numerators or only the denominators.
- use an incorrect equivalent fraction to compare the original fractions.
- confuse the symbols $<$ and $>$.
- incorrectly place fractions on a number line.

Lesson 18 Quiz

Solve the problems.

1 Choose *True* or *False* for each comparison. ***(2 points)***

	True	False
$\frac{8}{12} > \frac{5}{6}$	Ⓐ	Ⓑ
$\frac{3}{4} < \frac{3}{8}$	Ⓒ	Ⓓ
$\frac{2}{5} = \frac{4}{10}$	Ⓔ	Ⓕ
$\frac{6}{8} > \frac{1}{2}$	Ⓖ	Ⓗ
$\frac{2}{3} < \frac{2}{12}$	Ⓘ	Ⓙ

2 Devon walks $\frac{7}{12}$ of a mile to the library and $\frac{2}{3}$ of a mile to the store.

Which statement correctly describes a way to compare $\frac{7}{12}$ and $\frac{2}{3}$? ***(1 point)***

Ⓐ $2 < 7$, so $\frac{2}{3} < \frac{7}{12}$.

Ⓑ $\frac{2}{3} = \frac{8}{12}$, so $\frac{2}{3} > \frac{7}{12}$.

Ⓒ $12 > 3$, so $\frac{7}{12} > \frac{2}{3}$.

Ⓓ $\frac{7}{12} > \frac{1}{2}$ and $\frac{2}{3} < \frac{1}{2}$, so $\frac{7}{12} > \frac{2}{3}$.

Choice Matrix Scoring Rubric

2 points	1 point	0 points
All answers are correct	1 incorrect answer	2 or more incorrect answers

Fill-in-the-Blank Scoring Rubric

2 points	1 point	0 points
Response contains the following: • Correct computations, solutions, and/or calculations. • Demonstrates correct mathematical procedures and thinking.	Response contains the following: • Calculation error made. • Demonstrates mathematical procedures and/or thinking.	Response contains the following: • Incorrect answers that do not demonstrate the correct mathematical procedures and/or thinking.

Number Line Scoring Rubric

2 points	1 point	0 points
Response contains the following: • All fractions are correctly placed.	Response contains the following: • One fraction is incorrectly placed.	Response contains the following: • Fractions are incorrectly placed.

Extended Response Scoring Rubric

Points	Expectations
4	Response contains the following: • Correct solutions. (2 points) • Well-organized work and demonstration of thorough understanding of math concepts. (2 points)
3	Response contains the following: • Mostly correct solution(s). (1 point) • Shows strong understanding of math concepts. (2 points)
2	Response contains the following: • Shows partial to limited understanding of mathematical concepts.
1	Response contains the following: • Incorrect solution(s). • Poorly organized with incomplete work and explanations. • Demonstrates limited understanding of mathematical concepts.
0	Response contains the following: • No attempt to find a solution. • No effort to demonstrate an understanding of mathematical concepts.

Solutions

1 **B** (False); **D** (False); **E** (True); **G** (True); **J** (False)
2 points
4.NF.A.2, *DOK 2*

2 **B**; Students could solve the problem by multiplying the numerator and denominator of $\frac{2}{3}$ by 4 to recognize the equivalent fraction $\frac{8}{12}$, and correctly comparing $\frac{8}{12}$ and $\frac{7}{12}$.
A is not correct because only the numerators are compared.
C is not correct because only the denominators are compared.
D is not correct because although both fractions are greater than $\frac{1}{2}$, that is not enough information to correctly compare $\frac{7}{12}$ and $\frac{2}{3}$.
1 point
4.NF.A.2, *DOK 3*

3 See possible answers on the student page. Accept any true comparison that uses only the digits 2, 4, 6, or 8 once.
2 points
4.NF.A.2, *DOK 2*

4 **Part A**
See the number line on the student page.
2 points
4.NF.A.2, *DOK 1*

Part B
See possible explanation on the student page. Students may also multiply $\frac{4}{5}$ by 2 to find the equivalent fraction $\frac{8}{10}$, then compare $\frac{8}{10}$ to $\frac{3}{10}$.
4 points
4.NF.A.2, *DOK 3*

3 Use the digits 2, 4, 6, or 8 to make a true comparison. Each digit may be used once. Write the digits in the boxes. ***(2 points)***

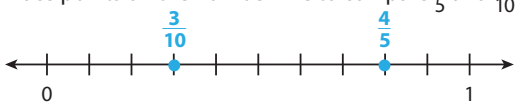
$\frac{5}{\square} < \frac{\square}{\square}$

Possible true comparisons: $\frac{5}{8} < \frac{4}{6}$; $\frac{5}{4} < \frac{8}{6}$; $\frac{5}{6} < \frac{8}{2}$

4 Compare $\frac{4}{5}$ and $\frac{3}{10}$.

Part A

Place points on the number line to compare $\frac{4}{5}$ and $\frac{3}{10}$. ***(2 points)***



Part B

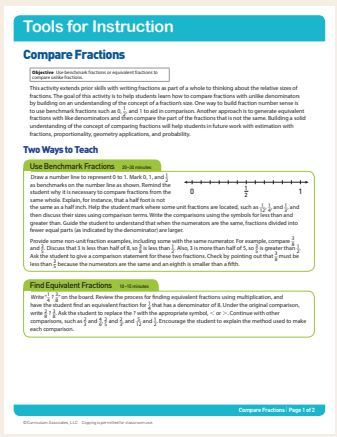
Write a comparison for $\frac{4}{5}$ and $\frac{3}{10}$. Explain how you found your answer. ***(4 points)***

$\frac{3}{10} < \frac{4}{5}$; Possible explanation: $\frac{4}{5}$ is greater than $\frac{3}{10}$ because $\frac{4}{5}$ is greater than $\frac{1}{2}$ and $\frac{3}{10}$ is less than $\frac{1}{2}$.

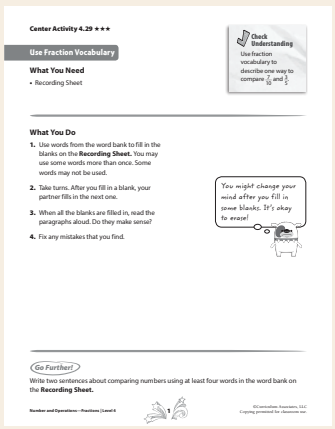
Differentiated Instruction

Teacher Toolbox

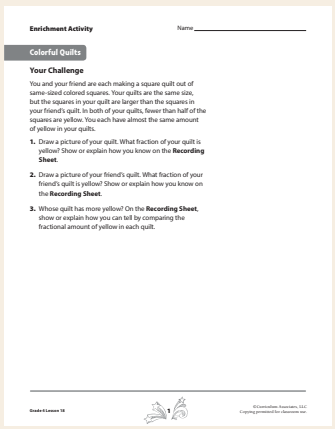
RETEACH: Tools for Instruction


Tools for Instruction
Compare Fractions
Two Ways to Teach
Use Benchmark Fractions
Find Equivalent Fractions

REINFORCE: Math Center Activities


Center Activity 4.29
Use Fraction Vocabulary
What You Need
What You Do
Go Further!

EXTEND: Enrichment Activities


Enrichment Activity
Colorful Quilts
Your Challenge